# Spatial Modeling of Device-to-Device Networks: Poisson Cluster Process Meets Poisson Hole Process

Mehrnaz Afshang and Harpreet S. Dhillon

***Abstract*—This paper combines Poisson Cluster Process (PCP) with a Poisson Hole Process (PHP) to develop a new spatial model for an *inband* device-to-device (D2D) communications network, where D2D and cellular transmissions share the same spectrum. The locations of the devices engaging in D2D communications are modeled by a modified Thomas cluster process in which the cluster centers are modeled by a PHP instead of more popular homogeneous Poisson Point Process (PPP). While the clusters capture the inherent proximity in the devices engaging in D2D communications, the holes model exclusion zones where D2D communication is prohibited in order to protect cellular transmissions. For this setup, we characterize network performance in terms of coverage probability and area spectral efficiency.**

## I. INTRODUCTION

The ability of devices to share content over direct wireless links, termed device-to-device communications, is a key component of the current 4G and future 5G wireless systems. Facilitated by the spatiotemporal correlation in the data demand, D2D allows *asynchronous content reuse* directly among wireless devices, thereby offloading traffic from cellular networks. For improved spectral utilization, D2D is usually envisioned to coexist with the cellular networks on the same frequency band, which is often termed as *inband* D2D communication. This naturally raises fundamental coexistence concerns, especially pertaining to the accurate interference characterization and management in the integrated D2D and cellular networks.

Any reasonable model for integrated D2D and cellular networks must capture at least two main aspects: (i) devices engaging in D2D communication should lie in close proximity of each other [1], and (ii) there must be spatial separation between cellular and inband D2D transmissions (for instance, by the creation of exclusion zones around cellular users) in order to protect cellular transmissions from excessive interference due to D2D transmission [2]–[5]. While there are works focusing on these two aspects separately, to the best of our knowledge their joint analysis is still an open problem. We recently captured the first aspect by envisioning the set of proximate devices as a *cluster* and hence modeling their locations by a Poisson Cluster Process (PCP) [6], [7]. The analysis was however focused on the out-of-band D2D, which means the second aspect did not come into picture.

Similarly, the analysis of second aspect of that of interference management does not generally incorporate device clustering [2]–[5]. The device locations are usually modeled by a Poisson Dipole Process (PDP), wherein transmitters are modeled as a homogeneous PPP, with the corresponding

The authors are with Wireless@VT, Department of ECE, Virgina Tech, Blacksburg, VA, USA. Email: {mehrnaz, hdhillon}@vt.edu.

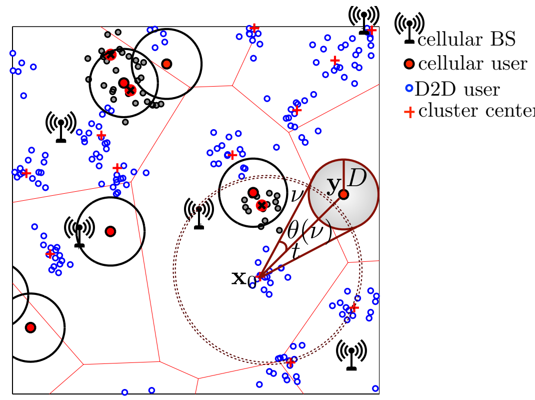


Fig. 1. Illustration of the proposed system model.

receivers located at a predefined fixed distance from each transmitter. Using PDP as a baseline process, the impact of interference management is usually captured by modeling the locations of D2D transmitters as a Poisson Hole Process (PHP) [8]. In particular, each hole (exclusion zone) of PHP corresponds to the region in which D2D transmitters are not allowed to reuse the cellular spectrum. While this model provides meaningful first order insights, it fails to capture the notion of device clustering, which as discussed above, is quite fundamental to D2D. Moreover, the analysis of PHP in all these works is typically based on the independent thinning of the PDP such that the resulting density of the thinned PDP is the same as that of the PHP. This approach may remove dominant interferers from the baseline process, which results in the overestimation of coverage probability. To address these shortcomings, we develop a new spatial model, where PCP and PHP are combined in a way that captures both the above mentioned aspects accurately. More details are given next.

*Contributions and outcomes:* We develop a new spatial model, where the downlink cellular and D2D networks coexist in the same band. In particular, we introduce a new cluster process in which the centers of *active* clusters (the ones that are allowed to transmit) are drawn from a PHP to protect cellular transmissions. This model will be henceforth referred to as a Hole Cluster Process (HCP). The key intermediate step in our analysis is the derivation of the Laplace transform of interference at a typical user of HCP. This analysis builds on our recent work on PHP [9], where we derived a new set of tight bounds on the Laplace transform of interference in a PHP. Using these results, we characterize the performance of the network in terms of the coverage probability and area spectral efficiency (ASE). This analysis leads to several system design guidelines. For instance, it reveals that there exists an *optimal* number of simultaneously active links per cluster (not necessarily one) that maximizes the network ASE.

## II. System Model

In this paper, we consider a D2D underlaid cellular downlink network where the locations of BSs are modeled as a homogeneous PPP $\{\mathbf{z}\} \equiv \Phi_{\rm b}$ with density $\lambda_{\rm b}$. We assume a saturated traffic model, where all the resource blocks of the BS are always scheduled. In other words, there is always one user scheduled on a typical resource block. Therefore, the locations of the cellular users being served on a typical resource block can be *reasonably* approximated by a homogeneous PPP $\{\mathbf{y}\} \equiv \Phi_{\rm u}$ with the same density as that of the BSs, i.e., $\lambda_{\rm b}$. Note that while the locations of these users and their serving BSs are correlated, ignoring this correlation usually results in fairly good approximations [10]. Therefore, $\Phi_{\rm u}$ is assumed to be independent of all other locations. In order to capture both the key aspects of inband D2D networks (device clustering and spatial separation), the D2D user locations are modeled by what we term as a *Hole Cluster Process (HPC)*. To construct HCP, we begin with a baseline homogeneous process $\Phi_{\rm c}$ of density $\lambda_{\rm c}$ of *cluster centers*. The cluster centers lying within a radius $D$ of the cellular users are *deactivated*, i.e., none of the users in these clusters will be allowed to transmit. The point process of *active* cluster centers can be expressed as

$$\Psi_{\rm c} = \{\mathbf{x} \in \Phi_{\rm c} : \mathbf{x} \notin \Xi_D\} = \Phi_{\rm c} \setminus \Xi_D, \tag{1}$$

where $\Xi_D \triangleq \bigcup_{\mathbf{y}\in\Phi_{\rm u}} \mathbf{b}(\mathbf{y}, D)$ with $\mathbf{b}(\mathbf{y}, D)$ being a ball of radius $D$ centered at $\mathbf{y}$. Now the D2D users are assumed to be independent and identically distributed (i.i.d.) around each cluster center with probability density function (PDF)

$$f_A(\mathbf{a}) = \frac{1}{2\pi\sigma^2} \exp\left(-\frac{\|\mathbf{a}\|^2}{2\sigma^2}\right), \quad \mathbf{a} \in \mathbb{R}^2. \tag{2}$$

If $D = 0$, the locations of cluster centers can be modeled by its baseline homogeneous PPP $\Phi_{\rm c}$, where this setup (i.e., $D = 0$) corresponds to the well-known Thomas cluster process [11].

Let us denote the set of transmitting (receiving) D2D users in cluster $\mathbf{x}$ by $\mathcal{N}_{\rm t}^{\mathbf{x}}$ ($\mathcal{N}_{\rm r}^{\mathbf{x}}$), where we assume that the subset of D2D users chosen uniformly at random, $\mathcal{A}^{\mathbf{x}} \subseteq N_{\rm t}$, are simultaneously transmitting within any given cluster. Therefore, the set of simultaneously transmitting (also referred to as *active*) D2D users in the whole network can be expressed as $\Psi_{\rm m} = \cup_{\mathbf{x}\in\Psi_{\rm c}} \mathcal{A}^{\mathbf{x}}$. To maintain generality, the number of simultaneously active transmitters per cluster, i.e., $|\mathcal{A}^{\mathbf{x}}|$, is modeled as a Poisson distributed random variable with mean $\bar{m}$. For this setup, we assume that all BSs and all D2D users transmit with power $P_{\rm c}$ and $P_{\rm d}$, respectively. Our main focus will be on the performance analysis of typical cellular and D2D users. More details are provided next.

*1) Typical D2D user:* It is a randomly chosen user from a randomly chosen cluster termed as a *representative cluster* centered at $\mathbf{x}_0 \in \Psi_{\rm c}$. We assume that the content of interest for this typical D2D user (located at the origin) is available with another D2D user chosen uniformly at random from the same cluster located at $\mathbf{x}_0 + \mathbf{a}_0$ [6]. Denoting the distance between the serving user and a typical D2D user by $r_{\rm d} = \|\mathbf{x}_0 + \mathbf{a}_0\|$, the received power at the typical user can be expressed as $P_{\rm d} h_{\rm d} r_{\rm d}^{-\alpha}$. Here, $h_{\rm d} \sim \exp(1)$ models Rayleigh fading and $\alpha > 2$ is the pathloss exponent. In this setup, the total interference experienced by a typical D2D user originates from three sources: (i) interference from simultaneously active D2D transmitters inside the representative cluster termed as *intra-cluster interference*, which is defined as $\mathcal{I}_{\tt d,d}^{\tt intra} = \sum_{\mathbf{a}\in\mathcal{A}^{\mathbf{x}_0}\setminus\mathbf{a}_0} P_{\rm d} h_{\rm d} \|\mathbf{x}_0 + \mathbf{a}\|^{-\alpha}$, (ii) interference from simultaneously active D2D transmitters outside the representative cluster termed as *inter-cluster interference*, which is defined as $\mathcal{I}_{\tt d,d}^{\tt inter} = \sum_{\mathbf{x}\in\Psi_{\rm c}\setminus x_0} \sum_{\mathbf{a}\in\mathcal{A}^{\mathbf{x}}} P_{\rm d} h_{\rm d} \|\mathbf{x}+\mathbf{a}\|^{-\alpha}$, and (iii) interference from cellular BSs defined as $\mathcal{I}_{\tt c,d} = \sum_{\mathbf{z}\in\Phi_{\rm b}} P_{\rm c} h_{\rm c} \|\mathbf{z}\|^{-\alpha}$. As a result, the signal to interference ratio (SIR) at the typical D2D user can be expressed as ${\tt SIR}_{\rm d} = \frac{P_{\rm d} h_{\rm d} r_{\rm d}^{-\alpha}}{\mathcal{I}_{\tt d,d}^{\tt intra} + \mathcal{I}_{\tt d,d}^{\tt inter} + \mathcal{I}_{\tt c,d}}$. In this paper, thermal noise is assumed to be negligible compared to the interference and is hence ignored.

*2) Typical cellular user:* A separate analysis will also be performed for a typical cellular user, which is simply a randomly chosen user from $\Phi_{\rm u}$ that can be placed at the origin due to stationarity of all the point processes involved. This user is assumed to connect to its closest BS located at $\mathbf{z}_0 \in \Phi_{\rm b}$. Denoting the distance between the typical cellular user located at the origin and its serving BS by $r_{\rm c} = \|\mathbf{z}_0\|$, the received power at the typical user is $P_{\rm c} h_{\rm c} r_{\rm c}^{-\alpha}$. In contrast with the typical D2D user discussed above, the total interference experienced by a typical cellular user originates from two sources: (i) interference from other cellular BSs (except the serving BS) defined as $\mathcal{I}_{\tt c,c} = \sum_{\mathbf{z}\in\Phi_{\rm b}\setminus\mathbf{z}_0} P_{\rm c} h_{\rm c} \|\mathbf{z}\|^{-\alpha}$, and (ii) interference from active D2D users defined as $\mathcal{I}_{\tt d,c} = \sum_{\mathbf{x}\in\Psi_{\rm c}} \sum_{\mathbf{a}\in\mathcal{A}^{\mathbf{x}}} P_{\rm d} h_{\rm d} \|\mathbf{x} + \mathbf{a}\|^{-\alpha}$. So, the SIR experienced by a typical cellular user is ${\tt SIR}_{\rm c} = \frac{P_{\rm c} h_{\rm c} r_{\rm c}^{-\alpha}}{\mathcal{I}_{\tt d,c} + \mathcal{I}_{\tt c,c}}$.

## III. Coverage Probability and ASE Analysis

This is the main technical section of this paper, where we first derive tight bounds on the Laplace transform of interference originated from HCP. Using these results, we will derive the coverage probability of typical cellular and D2D users, and ASE of the whole network.

### A. Coverage probability of a typical user of HCP

To fix key ideas upfront, we first characterize the performance of a typical user of HCP in the absence of the cellular network interference. This result will then be used to study the performance of typical D2D and typical cellular users. Before going into the detailed analysis of coverage probability, we first look at the distribution of the distances of serving and interfering devices to the typical user of HCP. Let us denote the set of distances from active D2D transmitters to the typical user inside the representative cluster by $\{s = \|\mathbf{x}_0 + \mathbf{a}\|\}$. These distances are correlated due to the common factor $\mathbf{x}_0$. So conditioning on $\nu_0 = \|\mathbf{x}_0\|$ along with the fact that $\{\mathbf{a}\}$ are i.i.d. zero-mean Gaussian random variables in $\mathbb{R}^2$, the set of distances $\{s = \|\mathbf{x}_0 + \mathbf{a}\|\}$ are conditionally i.i.d., where the PDF of each element is $f_S(s|\mathbf{x}_0) = f_S(s|\nu_0) = {\tt Ricepdf}(s, \nu_0; \sigma^2) = \frac{s}{\sigma^2} \exp\left(-\frac{s^2+\nu_0^2}{2\sigma^2}\right) I_0\left(\frac{s\nu_0}{\sigma^2}\right)$. Here, $I_0(.)$ is the modified Bessel function with order zero and $\sigma$ is the scale parameter. In [6], we showed that the "weaker" condition, i.e., conditioning on $\nu_0$ instead of $\mathbf{x}_0$, suffices. Recall that the selection of serving D2D user was done uniformly at random and hence the PDF of D2D serving distance, i.e., $R_\mathrm{d} = \|\mathbf{x}_0 + \mathbf{a}_0\|$, is $f_{R_\mathrm{d}}(r_\mathrm{d}|\nu_0) = \mathtt{Ricepdf}(r_\mathrm{d}, \nu_0; \sigma^2)$. The random selection of serving D2D user also implies that the PDF of the distance from intra-cluster interferer to a typical D2D user, i.e., $W = \|\mathbf{x}_0 + \mathbf{a}\|$, is $f_W(w|\nu_0) = \mathtt{Ricepdf}(w, \nu_0; \sigma^2)$. Using the same argument, the PDF of the distance between an inter-cluster interferer and a typical D2D user, i.e., $U = \|\mathbf{x} + \mathbf{a}\|$ conditioned on $\nu = \|\mathbf{x}\|$ is $f_U(u|\nu) = \mathtt{Ricepdf}(u, \nu; \sigma^2)$. Interested readers can refer to [6] for more details, where we provide a much more elaborate discussion about these distance distributions. Now using these distance distributions, the Laplace transform of intra-cluster interference is given in the next lemma. The proof follows on the same lines as that of [6, Lemma 5] and is hence skipped.

**Lemma 1.** *Laplace transform of intra-cluster interference,* $I^{\mathtt{intra}}_{\mathtt{d,d}} = \sum_{\mathbf{a}\in\mathcal{A}^{\mathbf{x}_0}} P_\mathrm{d} h_\mathrm{d} \|\mathbf{z}_0 + \mathbf{a}\|$, *conditioned on the distance* $\nu_0$, *is* $\mathcal{L}_{I^{\mathtt{intra}}_{\mathtt{d,d}}}(s|\nu_0) =$

$$\exp\left(-(\bar{m}-1)\int_0^\infty \frac{sP_\mathrm{d}w^{-\alpha}}{1+sP_\mathrm{d}w^{-\alpha}} f_W(w|\nu_0)\mathrm{d}w\right), \qquad (3)$$

*where* $f_W(w|\nu_0) = \mathtt{Ricepdf}(w, \nu_0; \sigma)$.

Characterizing Laplace transform of inter-cluster interference is much more challenging. This is mainly because the probability generating functional (PGFL) of cluster centers which are drawn from a PHP is unknown [8]. The most popular approach for the performance analysis of PHP is based on independent thinning of the baseline PPP $\Phi_\mathrm{c}$ such that the resulting density of the PPP is the same as that of a PHP [12]. Independent thinning of the baseline PPP disturbs the distribution of transmitters in the local neighborhood of the typical point, which results in a loose approximation of the Laplace transform of interference [9]. To address this shortcoming of PHP analysis, we recently developed a new approach in which the holes are preserved in the analysis in such a way that the resulting setup provides tight bounds on the Laplace transform of interference in a PHP [9]. We extend one of those results to a HCP here. We first consider the baseline PPP $\Phi_\mathrm{c}$ from which only one hole is carved out (at a given location). This setup is illustrated in Fig. 1, and the conditional Laplace transform of interference for this case is given next.

**Lemma 2.** *Let* $\mathcal{I} = \sum_{\mathbf{x}\in\{\Phi_\mathrm{c}\setminus\mathbf{x}_0\}\cap\mathbf{b}^c(\mathbf{y},D)} \sum_{\mathbf{a}\in\mathcal{A}^\mathbf{x}} P_\mathrm{d} h_\mathrm{d} \|\mathbf{x} + \mathbf{a}\|^{-\alpha}$. *The Laplace transform of interference* $\mathcal{I}$ *conditioned on* $t = \|\mathbf{y} - \mathbf{x}_0\|$ *is*

$$\mathcal{L}_{\mathcal{I}|t}(s) = \exp\left(-2\pi\lambda_\mathrm{c}\int_0^\infty \xi(sP_\mathrm{d},\nu)\nu\mathrm{d}\nu\right)\exp\left(2\lambda_\mathrm{c}\int_{t-D}^{t+D}\arccos\left(\frac{\nu^2+t^2-D^2}{2\nu t}\right)\xi(sP_\mathrm{d},\nu)\nu\mathrm{d}\nu\right) \text{ where,}$$

$$\xi(sP_\mathrm{d},\nu) = 1 - \exp\left(-\bar{m}\int_0^\infty \frac{sP_\mathrm{d}u^{-\alpha}}{1+sP_\mathrm{d}u^{-\alpha}} f_U(u|\nu)\mathrm{d}u\right), \quad (4)$$

*where* $f_U(u|\nu) = \mathtt{Ricepdf}(u, \nu; \sigma^2)$.

*Proof.* See Appendix A. ■

From Lemma 2, two bounds on the Laplace transform of inter-cluster interference are in order. First, we ignore the impact of the holes and approximate cluster center process PHP $\Psi_\mathrm{c}$ with its baseline PPP $\Phi_\mathrm{c}$. Note that this approach leads to the overestimation of the inter-cluster interference. As a result a lower bound on the Laplace transform of inter-cluster interference $\mathcal{I}^{\mathtt{inter}}_{\mathtt{d,d}}$ can be readily derived by substituting $D = 0$ in Lemma 2. The result is given in the next corollary.

**Corollary 1** (Lower bound 1)**.** *The Laplace transform of inter-cluster interference,* $\mathcal{I}^{\mathtt{inter}}_{\mathtt{d,d}}$, *is lower bounded by*

$$\mathcal{L}_{\mathcal{I}^{\mathtt{inter}}_{\mathtt{d,d}}}(s) \geq \exp\left(-2\pi\lambda_\mathrm{c}\int_0^\infty \xi(sP_\mathrm{d},\nu)\nu\mathrm{d}\nu\right), \quad (5)$$

*where* $\xi(sP_\mathrm{d},\nu)$ *is given by* (4).

Instead of completely ignoring the impact of holes, we now consider one hole in $\Psi_\mathrm{c}$; the one that is closest to the cluster center of the representative cluster. Denoting the location of this hole by $\mathbf{y}_0 \in \Phi_\mathrm{u}$, the interference field in this case is $\cup_{\mathbf{x}\in\{\Phi_\mathrm{c}\setminus\mathbf{x}_0\}\cap\mathbf{b}^c(\mathbf{y}_0,D)}\mathcal{A}^\mathbf{x} \supset \Psi_\mathrm{m}$, which clearly overestimates the true interference and hence leads to another, slightly tighter, lower bound on the Laplace transform of inter-cluster interference. If the distance between the closest hole to the center of representative cluster is $t_0 = \|\mathbf{y}_0 - \mathbf{x}_0\|$, then the PDF of $T_0$ (where $t_0$ is realization of $T_0$) is

$$f_{T_0}(t_0) = 2\pi\lambda_\mathrm{b} t_0 \exp\left(-\pi\lambda_\mathrm{b}\left(t_0^2 - D^2\right)\right), \quad t_0 > D. \quad (6)$$

This PDF can be simply derived based on the fact that the closest hole to the center of representative cluster lies outside $\mathbf{b}(\mathbf{x}_0, D)$ [9]. Now, deconditioning the results of Lemma 2 with respect to $T_0$, the second lower bound on the Laplace transform of inter-cluster interference $\mathcal{I}^{\mathtt{inter}}_{\mathtt{d,d}}$ is derived next.

**Corollary 2** (Lower bound 2)**.** *The Laplace transform of inter-cluster interference* $\mathcal{I}^{\mathtt{inter}}_{\mathtt{d,d}}$ *is lower bounded by* $\mathcal{L}_{\mathcal{I}^{\mathtt{inter}}_{\mathtt{d,d}}}(s) \geq$

$$\exp\left(-2\pi\lambda_\mathrm{c}\int_0^\infty \xi(sP_\mathrm{d},\nu)\nu\mathrm{d}\nu\right)\int_D^\infty \exp\left(2\lambda_\mathrm{c}\int_{t_0-D}^{t_0+D}\arccos\left(\frac{\nu^2+t_0^2-D^2}{2\nu t_0}\right)\xi(sP_\mathrm{d},\nu)\nu\mathrm{d}\nu\right) f_{T_0}(t_0)\mathrm{d}t_0, \quad (7)$$

*where* $f_{T_0}(t_0)$ *is given by* (6).

Using the bounds derived in Corollaries 1 and 2, we now derive two bounds on the coverage probability of the typical user in an HCP. Coverage probability can be formally defined as the probability that $\mathtt{SIR}$ experienced by a typical user is greater than predetermined threshold $\beta$ for successful reception. Denoting the serving distance by $R$, the coverage probability can be mathematically expressed as $\mathtt{P_c} = \mathbb{E}_R[\mathbb{P}(\mathtt{SIR}(R) > \beta|R)]$. Rayleigh fading assumption along with the fact that intra- and inter-cluster interferers are independent allows us to characterize coverage probability of a typical user as a product of Laplace transform of intra- and inter-cluster interference, where formal proof is similar to the proof of [6, Theorem 1]. Now, using this definition the coverage probability of a typical user of HCP is stated next.

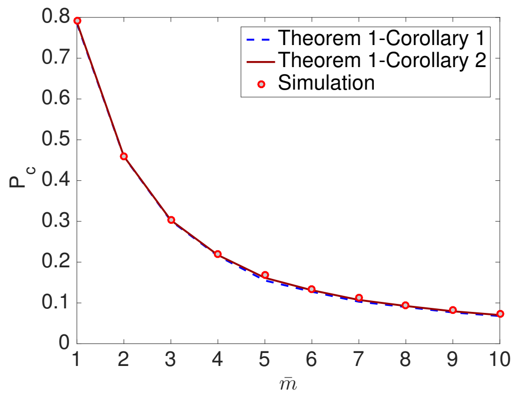


Fig. 2. Validation of bounds on HCP when $\beta = 0$ dB, $\alpha = 4$, $P_c = 200P_d$, $\lambda_c = 150$ cluster/ km$^2$, $\lambda_c = 5\lambda_b$, and $\sigma = 10$. Corollaries in the legend correspond to the bounds on the Laplace transform of inter-cluster interference.

**Theorem 1.** *Coverage probability of a typical user of HCP in the absence of interference from the cellular network is*

$$\mathtt{P_c} = \int_0^\infty \int_0^\infty \mathcal{L}_{I_{\mathrm{d,d}}^{\mathtt{intra}}}\left(\frac{\beta}{P_\mathrm{d}} r_\mathrm{d}^\alpha | \nu_0\right) \mathcal{L}_{\mathcal{I}_{\mathrm{d,d}}^{\mathtt{inter}}}\left(\frac{\beta}{P_\mathrm{d}} r_\mathrm{d}^\alpha\right) f_{R_\mathrm{d}}(r_\mathrm{d}|\nu_0) f_{V_0}(\nu_0) \mathrm{d}r_\mathrm{d}\mathrm{d}\nu_0, \text{ where} \quad (8)$$

$f_{V_0}(\nu_0) = \frac{\nu_0}{\sigma^2}\exp\left(-\frac{\nu_0^2}{2\sigma^2}\right)$, *and* $\mathcal{L}_{I_{\mathrm{d,d}}^{\mathtt{intra}}}(.)$ *is given by Lemma 1. Two lower bounds on* $\mathtt{P_c}$ *can be derived by substituting the results of Corollaries 1 and 2 for* $\mathcal{L}_{\mathcal{I}_{\mathrm{d,d}}^{\mathtt{inter}}}(.)$ *above.*

As illustrated in Fig. 2, the two bounds derived in the above Theorem are surprisingly tight. This is because of the fact that the coverage probability strongly depends on the set of interfering users that are in close proximity to the typical user, in particular the intra-cluster interferers, which have been accurately accounted for in the bounds. With this insight, we will use the lower bound on the Laplace transform of inter-cluster interference given by Corollary 1 as a proxy of the exact expression in the rest of this paper to maintain analytical tractability. Clearly, this will not result in any loss of accuracy.

### B. Coverage probability of D2D user

We now extend the discussion from the previous subsection to study the performance of a typical D2D user in terms of coverage probability. In addition to the interference components considered already, we need to derive the Laplace transform of interference originating from the cellular network, which is done in the next Lemma.

**Lemma 3.** *Laplace transform of interference,* $\mathcal{I}_{\mathtt{c,d}} = \sum_{\mathbf{z}\in\Phi_\mathrm{b}} P_\mathrm{c} h_\mathrm{c} \|\mathbf{z}\|^{-\alpha}$, *is*

$$\mathcal{L}_{\mathcal{I}_{\mathrm{c,d}}}(s) = \exp\left(-\pi\lambda_\mathrm{b}(sP_\mathrm{c})^{2/\alpha}\frac{2\pi/\alpha}{\sin(2\pi/\alpha)}\right). \quad (9)$$

The proof follows on the same line as proof of Laplace transform of interference in [8, Section: 5.1.7] and is hence skipped here due to space constraints. Combining all these results, the coverage probability of a typical D2D user is formally stated in the next theorem.

**Theorem 2.** *Coverage probability of a typical D2D user is*

$$\mathtt{P}_\mathtt{c}^{(\mathtt{d})} = \int_0^\infty \int_0^\infty \mathcal{L}_{\mathcal{I}_{\mathrm{d,d}}^{\mathtt{intra}}}\left(\frac{\beta}{P_\mathrm{d}} r_\mathrm{d}^\alpha | \nu_0\right) \mathcal{L}_{\mathcal{I}_{\mathrm{d,d}}^{\mathtt{inter}}}\left(\frac{\beta}{P_\mathrm{d}} r_\mathrm{d}^\alpha\right) \mathcal{L}_{\mathcal{I}_{\mathrm{c,d}}}\left(\frac{\beta}{P_\mathrm{d}} r_\mathrm{d}^\alpha\right) f_{R_\mathrm{d}}(r_\mathrm{d}|\nu_0) f_{V_0}(\nu_0) \mathrm{d}r_\mathrm{d}\mathrm{d}\nu_0, \text{ with} \quad (10)$$

$f_{V_0}(\nu_0) = \frac{\nu_0}{\sigma^2}\exp\left(-\frac{\nu_0^2}{2\sigma^2}\right)$, *where* $\mathcal{L}_{\mathcal{I}_{\mathrm{d,d}}^{\mathtt{intra}}}(.)$ *and* $\mathcal{L}_{\mathcal{I}_{\mathrm{c,d}}}(.)$ *are given by* (3), *and* (9). *We use the lower bound for* $\mathcal{L}_{\mathcal{I}_{\mathrm{d,d}}^{\mathtt{inter}}}(.)$ *given by* (5), *which results in a lower bound on* $\mathtt{P}_\mathtt{c}^{(\mathtt{d})}$.

### C. Coverage probability of cellular user

Recall that a typical cellular user is served by the nearest BS from $\Phi_\mathrm{b}$. Denoting the cellular serving distance by $R_\mathrm{c} = \|\mathbf{z}_0\|$, the PDF of $R_\mathrm{c}$ is $f_{R_\mathrm{c}}(r_\mathrm{c}) = 2\pi\lambda_b r_\mathrm{c}\exp(-\lambda_\mathrm{b}\pi r_\mathrm{c}^2)$ [8]. Hence, the aggregate interference power at a typical cellular user (located at the origin) is formed by the contribution of all cellular interferers which lie outside $\mathbf{b}(0, r_\mathrm{c})$. Here, the Laplace transform of interference $\mathcal{I}_{\mathtt{c,c}}$, can be derived by using standard arguments as in [8, Section: 5.1.7]. The result is stated next and the proof is skipped.

**Lemma 4.** *Laplace transform of interference* $\mathcal{I}_{\mathtt{c,c}} = \sum_{\mathbf{z}\in\Phi_\mathrm{b}\setminus\mathbf{z}_0} P_\mathrm{c} h_\mathrm{c} \|\mathbf{z}\|^{-\alpha}$, *is* $\mathcal{L}_{\mathcal{I}_{\mathrm{c,c}}}(s) =$

$$\exp\left(-\pi\lambda_\mathrm{b}(sP_\mathrm{c})^{2/\alpha}\int_{r^2/(sP_\mathrm{c})^{2/\alpha}}^\infty \frac{1}{1+u^{\alpha/2}}\mathrm{d}u\right). \quad (11)$$

The second component of interference in this case is the interference originating from the D2D network. To characterize this interference, note that the cluster center of the *active* clusters lie outside $\mathbf{b}(0, D)$ with respect to the typical cellular user due to the formation of the exclusion zones (holes) around cellular users. The Laplace transform of interference from D2D networks to a typical cellular user is given next, where the proof follows on the same line as that of Corollary 1.

**Corollary 3.** *The Laplace transform of interference,* $\mathcal{I}_{\mathtt{d,c}}$, *is lower bounded by*

$$\mathcal{L}_{\mathcal{I}_{\mathrm{d,c}}}(s) \geq \exp\left(-2\pi\lambda_\mathrm{c}\int_D^\infty \xi(sP_\mathrm{d},\nu)\nu\mathrm{d}\nu\right), \quad (12)$$

*where* $\xi(sP_\mathrm{d},\nu)$ *is given by* (4).

Combining these results, the coverage probability of the typical cellular user is stated next.

**Theorem 3.** *Coverage probability of a typical cellular user is*

$$\mathtt{P}_\mathtt{c}^{(\mathtt{c})} = \int_0^\infty \mathcal{L}_{\mathcal{I}_{\mathrm{d,c}}}\left(\frac{\beta}{P_\mathrm{c}} r_\mathrm{c}^{-\alpha}\right) \mathcal{L}_{\mathcal{I}_{\mathrm{c,c}}}\left(\frac{\beta}{P_\mathrm{c}} r_\mathrm{c}^{-\alpha}\right) f_{R_\mathrm{c}}(r_\mathrm{c})\mathrm{d}r_\mathrm{c}, \quad (13)$$

*where* $\mathcal{L}_{\mathcal{I}_{\mathrm{c,c}}}(.)$ *and* $\mathcal{L}_{\mathcal{I}_{\mathrm{d,c}}}(.)$ *are given by* (11) *and* (12).

### D. Area Spectral efficiency

We now focus on the average number of bits transmitted per unit time per unit bandwidth per unit area, termed as *area spectral efficiency* ($\mathtt{ASE}$). It is mathematically defined as $\mathtt{ASE} = \lambda\log_2(1+\beta)\mathbb{E}[\mathbf{1}\{\mathtt{SIR}(r) > \beta\}]$, where $\lambda$ is the density of transmitters. We specialize this definition for our setup in the next Proposition. The proof is provided in Appendix B.

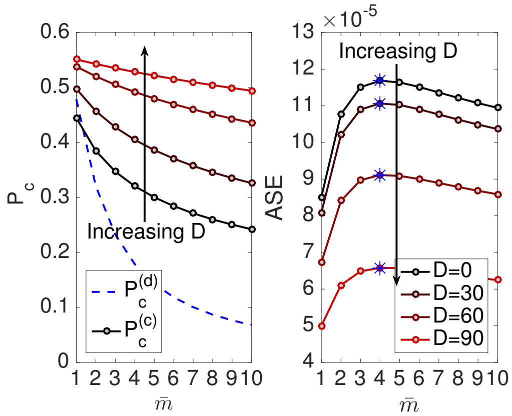


Fig. 3. $\mathtt{P_c}$ and $\mathtt{ASE}$ when $\beta = 0$ dB, $\alpha = 4$, $P_c = 200P_d$, $\lambda_c = 150$ cluster/km$^2$, $\lambda_c = 5\lambda_b$, and $\sigma = 10$.

**Proposition 1.** *The* ASE *of the network is,*

$$\mathtt{ASE} = [\bar{m}\lambda_c \exp(-\pi\lambda_b D^2)\mathtt{P}_c^{(d)} + \lambda_b \mathtt{P}_c^{(c)}] \log_2(1+\beta) \quad (14)$$

*where* $\mathtt{P}_c^{(d)}$ *and* $\mathtt{P}_c^{(c)}$ *are given by* (10) *and* (13)*, respectively. Here,* $\bar{m}\lambda_c \exp(-\pi\lambda_b D^2)$ *is the average number of simultaneously active D2D links and* $\lambda_b$ *is the average number of simultaneously active cellular links per unit area.*

Coming to the design insights, note that there exists a fundamental tradeoff between (i) number of simultaneously active D2D transmitters, and (ii) exclusion zone radius and resulting interference. While aggressive frequency reuse (i.e., decreasing exclusion zone radius and/or increasing number of simultaneously active D2D transmitters) potentially increases ASE, it also increases interference significantly. To study this tradeoff, we plot the coverage probability and the ASE with respect to the number of simultaneously active D2D transmitters $\bar{m}$ for different values of exclusion zone radius $D$ in Fig. 3. For this setup, it can be seen that reducing $D$ increases ASE. Interestingly, the optimum number of simultaneously active transmitters per cluster remains the same and equal to $\bar{m}^* = 4$. This is because optimal value of $\bar{m}$ is mainly dictated by intra-cluster interference which is not a function of $D$.

## IV. Conclusion

In this paper, we developed a comprehensive framework for the analysis of inband D2D communications. Modeling the D2D network as a HCP, we captured (i) the inherent proximity in the devices engaging in D2D communications, and (ii) the spatial separation between active cellular and D2D links. For this setup, we derived tight bounds on the Laplace transform of interference originating from HCP. Using these bounds, the coverage probability and ASE were accurately characterized.

## Appendix

### A. Proof of Lemma 2

Defining $\Omega = \{\Phi_c \setminus \mathbf{x}_0\} \cap \mathbf{b}^c(\mathbf{y}, D)$, the Laplace transform of interference conditioned on $t = \|\mathbf{y} - \mathbf{x}_0\|$ is $\mathcal{L}_{\mathcal{I}|t}(s) = \mathbb{E}\Big[\exp\Big(-s\sum_{\mathbf{x}\in\Omega}\sum_{\mathbf{a}\in\mathcal{A}^{\mathbf{x}}} P_d h_d \|\mathbf{x}+\mathbf{a}\|^{-\alpha}\Big)\Big]$

$$\stackrel{(a)}{=} \mathbb{E}_{\Phi_c}\Big[\prod_{\mathbf{x}\in\Omega} \exp\Big(-\bar{m}\int_0^\infty \frac{1}{1+u^\alpha/(sP_d)} f_U(u|\|\mathbf{x}\|)\mathrm{d}u\Big)\Big]$$

$$\stackrel{(b)}{=} \exp\Big(-\lambda_c \int_{\mathbb{R}^2\setminus\mathbf{b}(\mathbf{y},D)} \xi(sP_d, \|\mathbf{x}\|)\mathrm{d}\mathbf{x}\Big)$$

$$\stackrel{(c)}{=} \exp\Big(-2\pi\lambda_c \int_0^\infty \xi(sP_d,\nu)\nu\mathrm{d}\nu\Big)\exp\Big(2\lambda_c \int_{t-D}^{t+D} \arccos\Big(\frac{\nu^2+t^2-D^2}{2\nu t}\Big)\xi(sP_d,\nu)\nu\mathrm{d}\nu\Big)$$

where $(a)$ follows from $h \sim \exp(1)$, along with expectation over number of active transmitting users which are Poisson distributed, $(b)$ from PGFL of PPP [8], where $\xi(sP_d, \nu)$ is given by (4), and $(c)$ from the cosine-law: $\nu^2 + t^2 - 2\nu t\cos\theta(r) = D^2$ (Fig. 1) along with converting form Cartesian to polar coordinates by substituting $\nu = \|\mathbf{x}\|$.

### B. Proof of Proposition 1

The proof simply follows from definition of ASE, where the number of active D2D transmitters $\bar{m}_{\mathtt{HCP}}$ can be derived as

$$\bar{m}_{\mathtt{HCP}} \stackrel{(a)}{=} \mathbb{E}\Big[\sum_{\mathbf{x}\in\Phi_c}\sum_{\mathbf{a}\in\mathcal{A}^{\mathbf{x}}}\prod_{\mathbf{y}\in\Phi_u}(1-\mathbf{1}_{\mathbf{b}(\mathbf{x},D)}(\mathbf{y}))\Big]$$

$$\stackrel{(b)}{=} \mathbb{E}\Big[\sum_{\mathbf{x}\in\Phi_c}\sum_{\mathbf{a}\in\mathcal{A}^{\mathbf{x}}}\exp(-\lambda_b\int_{\mathbb{R}^2}\mathbf{1}_{\mathbf{b}(\mathbf{x},D)}(\mathbf{y})\mathrm{d}\mathbf{y})\Big] = \lambda_c\bar{m}e^{-\lambda_b\pi D^2},$$

where $(a)$ follows from the fact that by definition there are no cellular users $\mathbf{y} \in \Phi_u$ close to the active cluster centers (i.e., $1-\mathbf{1}_{\mathbf{b}(\mathbf{x},D)}(\mathbf{y})$), and $(b)$ follows from Campbell's theorem [8].

## References


[1] N. Golrezaei, A. F. Molisch, A. G. Dimakis, and G. Caire, "Femtocaching and device-to-device collaboration: A new architecture for wireless video distribution," *IEEE Commun. Magazine*, vol. 51, no. 4, pp. 142–149, Apr. 2013.

[2] H. Sun, M. Wildemeersch, M. Sheng, and T. Q. Quek, "D2D enhanced heterogeneous cellular networks with dynamic TDD," to appear, *IEEE Trans. on Wireless Commun.*, 2015, available online: arxiv.org/abs/1406.2752.

[3] G. George, R. K. Mungara, and A. Lozano, "An analytical framework for device-to-device communication in cellular networks," 2014, available online: arxiv.org/abs/1407.2201.

[4] A. H. Sakr and E. Hossain, "Cognitive and energy harvesting-based D2D communication in cellular networks: Stochastic geometry modeling and analysis," *IEEE Trans. on Commun.*, vol. 63, no. 5, pp. 1867–1880, May. 2015.

[5] R. K. Mungara, X. Zhang, A. Lozano, and R. W. Heath Jr., "On the spatial spectral efficiency of ITLinQ," in *Proc., IEEE Asilomar*, Pacific Grove, CA, Nov. 2014.

[6] M. Afshang, H. S. Dhillon, and P. H. J. Chong, "Modeling and performance analysis of clustered device-to-device networks," submitted to *IEEE Trans. on Wireless Commun.*, 2015, available online: arxiv.org/abs/1508.02668.

[7] ——, "Fundamentals of cluster-centric content placement in cache-enabled device-to-device networks," submitted to *IEEE Trans. on Commun.*, 2015, available online:arxiv.org/abs/1509.04747.

[8] M. Haenggi, *Stochastic Geometry for Wireless Networks*. Cambridge University Press, 2012.

[9] Z. Yazdanshenasan, H. S. Dhillon, M. Afshang, and P. H. J. Chong, "Poisson hole process: Theory and applications to wireless networks," submitted to *IEEE Trans. on Wireless Commun.*, 2015.

[10] T. D. Novlan, H. S. Dhillon, and J. G. Andrews, "Analytical modeling of uplink cellular networks," *IEEE Trans. on Wireless Commun.*, vol. 12, no. 6, pp. 2669–2679, Jun. 2013.

[11] R. K. Ganti and M. Haenggi, "Interference and outage in clustered wireless ad hoc networks," *IEEE Trans. on Info. Theory*, vol. 55, no. 9, pp. 4067–4086, Sep. 2009.

[12] C.-h. Lee and M. Haenggi, "Interference and outage in Poisson cognitive networks," *IEEE Trans. on Wireless Commun.*, vol. 11, no. 4, pp. 1392–1401, Apr. 2012.