Cognitive MIMO Radio: Incorporating Dynamic Spectrum Access in Multiuser MIMO Network

Harpreet S. Dhillon and R. Michael Buehrer
Wireless@Virginia Tech, Bradley Department of Electrical and Computer Engineering,
Virginia Polytechnic Institute and State University, Blacksburg, VA 24061, USA.
E-mail: {harpreet.dhillon, buehrer}@vt.edu

***Abstract*—In this paper, we develop a general mathematical framework to incorporate dynamic spectrum access in a multiuser MIMO network. This framework is particularly helpful in computing the maximum achievable system capacity of a resulting multiple-band multiuser MIMO network. The mathematical formulation to maximize the system capacity is shown to be quite similar to that of a well studied single-band multiuser MIMO network. It is further shown that the capacity maximization problem is equivalent to finding the optimal eigenvalues of the input symbol covariance matrices of the users in each frequency band. Due to the dependence of the eigenvalues on the physical characteristics of the system, such as orientation of the antennas and the channel conditions, it is difficult to achieve their optimal values in general. Because of this difficulty in achieving the optimal capacity, we also consider the suboptimal MIMO techniques (specifically beamforming) and study their capacity performance in a multiple-band multiuser MIMO system.**

***Index Terms*—Cognitive radio, dynamic spectrum access, dynamic spectrum allocation, multiuser MIMO network, beamforming, non-convex non-linear programming.**

I. Introduction

THE most important goals in spectrum assignment are avoiding interference and maximizing utilization. The traditional solution to this problem is to divide the spectrum into several non-overlapping frequency bands and assign each band to a wireless user/technology. Though this static assignment avoids interference between systems, it doesn't maximize spectrum utilization. Federal Communications Commission (FCC) measurements have clearly indicated that a significant number of licensed bands remain unused or underutilized for more than 90% of the time [1]. The high variability in the spectrum usage over frequency, time and space has attracted the interest of wireless communications community to develop efficient spectrum management methods. This has led to a flurry of research activity around the concept of dynamically utilizing the available spectrum [2]-[4]. The general idea is to equip a wireless device with "cognitive" capabilities so that it can adapt to the changing electromagnetic environment in order to maximize the utilization of the available resources [5]. Many terms are coined for variants of this basic idea which are well explained in [2]. In this paper, we specifically assume the overlay or the opportunistic dynamic spectrum access approach [5].

The opportunistic dynamic spectrum access approach (referred henceforth as DSA) is a hierarchical approach where the spectrum is available to a secondary user (SU) only when the primary user (licensee) is not transmitting in that frequency band. The main concern in DSA is to first identify the available frequency bands by sensing the spectrum and then to allocate them to the SUs. In this paper, we assume that we have perfect knowledge of the available frequency bands and will focus on spectrum sharing between SUs. Interested readers can refer to [6] for a discussion of various spectrum sensing techniques and to [7] for the benefit of cooperative spectrum sensing in cognitive radios. After identifying the available frequency bands, these are then allocated to various SUs with two primary goals: maximum spectrum utilization and fairness. These two goals lead to a number of utility (cost) functions that can be optimized for specific network applications. Some cost functions concentrate more on maximizing the utilization and others give more emphasis on fairness. Some examples of these cost functions for single-antenna systems can be found in [8], [9].

The problem of dynamic spectrum allocation to achieve the globally optimal system capacity in a single antenna system is known to be NP-hard [10]. In this case, a centralized server obtains information about the topology along with the user demand and assigns frequency bands to various SUs in order to maximize the spectral utilization. If the topology and user demand is fixed, these calculations can be performed only once to obtain conflict-free spectrum assignments that closely approximate the global optimum. If however, they are variable, a centralized system will have to find the optimal frequency allocation after each change, which adds significant computational and communication overhead. To overcome this problem, several central and distributed suboptimal spectrum allocation techniques are discussed in the literature [11]-[13]. In some distributed techniques, each user tries to optimize its own utility function and requires no collaboration from other users. In other distributed techniques, users group themselves into small groups based on geography or similarities in the technologies being used and optimize the spectrum allocation within the group to approximate the global optimum.

As is evident from the above discussion, the problem of optimal frequency allocation for SUs employing a single antenna is well-defined and solved from different perspectives in the literature [2]-[13]. The extension of this problem from single-antenna systems to multiple-antenna systems is not straight forward because of the presence of an additional degree of freedom. The global optimum of system capacity in this case can be achieved only when both the spectral and spatial (due to the presence of multiple-antennas) domains are optimized simultaneously. It should be noted that in the most general system model, time is also considered as a transmit degree of freedom in addition to space and frequency. It is not taken into consideration in the present discussion because we are assuming that all the SUs are concurrently transmitting.

The problem of incorporating DSA in a multiuser MIMO network is not well studied and is the main focus of this paper. We specifically develop a complete mathematical formulation to compute the maximum achievable capacity of a Multiple Band Multiuser MIMO (referred henceforth as MBMM) network. By multiple band system, we mean that multiple frequency bands are available for the SUs. We further show that this problem is equivalent to finding the optimal eigenvalues of the input symbol covariance matrices of the SUs in each frequency band. Due to the dependence of the eigenvalues on the physical characteristics of the system, such as orientation of the antennas and the channel conditions, it is difficult to achieve their optimal values in general. Because of this difficulty, it is necessary to investigate the system capacity of MBMM networks for sub-optimal MIMO techniques. This will also help us in identifying techniques that closely approximate the optimal capacity in given channel conditions. In particular, in this paper we study the sub-optimal approach of beamforming. We also show that the mathematical formulation of the DSA problem for MBMM networks is quite similar to that of the Single Band Multiuser MIMO (referred henceforth as SBMM) networks. This fact is quite important because it allows us to easily extend the results and algorithms proposed for the well-studied SBMM systems to cognitive MIMO systems.

In [14], it is shown that the sum of the mutual information of SUs in a SBMM network is neither a convex nor a concave function. It is thus very difficult to find the global maximum of this function analytically. Many local optimization techniques have been proposed in the literature to solve this problem [15], [16]. Though these approaches can quickly find a locally optimal solution, they cannot guarantee the global optimum for non-convex problems. Recently, a global optimization method was proposed to solve this problem for SBMM networks [17]. This method is based on the branch and bound framework, coupled with the reformulation-linearization technique and guarantees a globally optimal solution [18]. Because of the established similarity in the mathematical formulations, the algorithm can be extended (with suitable modifications) to compute the optimal system capacity of a MBMM network. The working of this algorithm is briefly explained in a later section.

The present problem of finding the maximum achievable capacity is also important because it provides a benchmark for evaluating the performance of decentralized techniques for incorporating DSA in MBMM networks. One such decentralized technique is proposed in [20], where each SU utilizes a transmission strategy which maximizes its own utility function. In the multiuser case, each SU will dynamically react to the strategies adopted by other users and the equilibrium achieved is shown to coincide with the Nash equilibrium [20].

The remainder of this paper is organized as follows. In Section II, the system model is explained along with all of the underlying assumptions. The problem of incorporating DSA in multiuser MIMO is discussed in Section III. Specifically, we develop a mathematical formulation of the optimization problem to maximize the capacity and show that it is quite similar to that of the SBMM network. Section IV deals with the problem of maximizing the system capacity when the SUs are employing beamforming both at the transmitter and receiver. Numerical results are presented in Section V. The paper is concluded in Section VI.

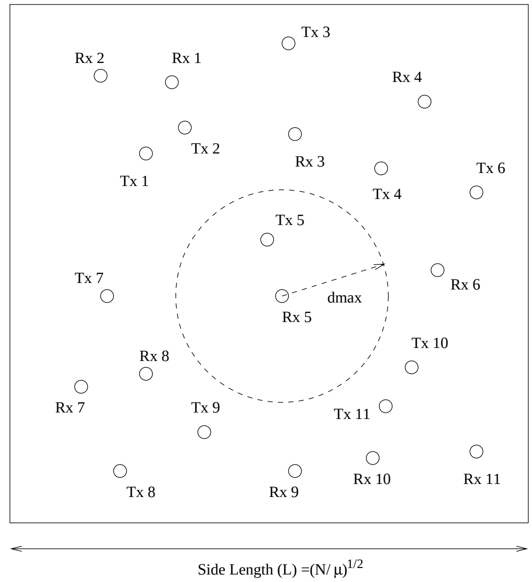

Fig. 1. System model depicting various Tx-Rx pairs.

II. System Model

A. Assumptions

Several assumptions are made in the analysis to facilitate the system layout. Firstly, it is assumed that no primary user is present in the frequency bands of interest and they are thus available for allocation to the SUs. Secondly, we assume that all the SUs consist of a transmitter (Tx) comprising of n_t transmitter antennas and a receiver (Rx) comprising of n_r receive antennas. In this peer-peer network model each Tx has only one intended Rx and acts as an interferer for the rest of the SUs. Further, we make the Gaussian interference channel assumption in which interference and noise is modeled as being Gaussian distributed. An example of the basic system setup is shown in Fig. 1. Thirdly, we assume that the available spectrum is divided into a countable number of orthogonal frequency bands. SUs are not restricted to transmit over a single band and can distribute transmit power over multiple frequency bands. Each Tx is assumed to have a total maximum transmit power of P_{max} over all frequency bands and all n_t transmit antennas. It is further assumed that all the users are sharing complete information and the system is centrally optimized to find the maximum sum of the mutual information. In this case, a centralized server obtains information about the topology and determines the optimal power allocation across both antennas and frequency bands for all SUs.

B. System Layout

The system layout is shown in Fig. 1. The maximum allowable distance between a Tx/Rx pair is denoted by d_{max}. d_{max} is assumed to be a system constant and is defined such that the minimum average received SNR per receive antenna is 1 dB. The density of the SUs is handled by defining a Multi-User Interference (MUI) factor. MUI represents the expected number of Rx nodes within a circle of radius d_{max} centered at any Tx node assuming a constant density. Increasing the MUI factor increases the density of interferers and hence increases the mutual interference. For a fixed MUI and d_{max} value, the density (μ) in terms of users per unit area can be evaluated as: $\mu = MUI/\pi d_{max}^2$. The complete area of interest is assumed to be a square. To place N users within a square with density μ, the square should have an area of $\frac{N}{\mu}$, and hence a side length of $\sqrt{\frac{N}{\mu}}$. For each analysis, we place a number of Rx units uniformly distributed in the chosen square area. Each Tx is then placed in the circle of radius d_{max} centered at the corresponding Rx, as shown in Fig. 1.

C. Mathematical Notations

Boldface is used to denote matrices and vectors. For a matrix $\mathbf{A}$, $\mathbf{A}^\dagger$ denotes the conjugate transpose and $\mathbf{A}^T$ denotes the transpose. $Tr\{\mathbf{A}\}$ denotes the trace of the matrix $\mathbf{A}$. $\mathbf{I}$ denotes the identity matrix, whose dimensions can be determined from the context. $\mathbf{A} \succ \mathbf{0}$ represents that $\mathbf{A}$ is Hermitian and positive semidefinite. The scalar $a_{m,n}$ represents the entry in the m^{th}-row and n^{th}-column of $\mathbf{A}$. For a complex scalar a, $\Re(a)$ and $\Im(a)$ represent the real and imaginary parts of a, respectively, and a' represents the conjugate of a. diag$\{\mathbf{A}\}$ denotes a vector of the diagonal elements of $\mathbf{A}$ and diag$\{\mathbf{A}\} \succeq \mathbf{0}$ means that all the diagonal elements of $\mathbf{A}$ are non-negative.

III. DSA in Multiuser MIMO: Maximum Capacity

Our goals in this section are to incorporate DSA in multiuser MIMO and to formulate a mathematical problem to determine the maximum sum of the mutual information of these interfering SUs.

A. Defining the Variables

We consider a network consisting of N mutually interfering SUs, which are indexed by $1, 2, \ldots, N$. In this analysis, it is assumed that the transmitters have full channel state information. Let the available spectrum be divided into m frequency bands, indexed by $1, 2, \ldots, m$. Let us denote the MIMO link from the Tx of the j^{th} SU to the Rx of the i^{th} SU to be L_{ji}. Let the matrix $\mathbf{H}_{ji}^l \in C^{n_r \times n_t}$ denote the channel matrix of link L_{ji} in the l^{th} frequency band. Let the matrix $\mathbf{Q}_i^l$ be the covariance matrix of the zero mean Gaussian transmit symbol vector $\mathbf{x}_i^l$ of the i^{th} SU in l^{th} frequency band, i.e., $\mathbf{Q}_i^l = E\{\mathbf{x}_i^l.\mathbf{x}_i^{l\dagger}\}$. Further denote ρ_{ji}^l as the signal-to-noise ratio per unit transmit power in frequency band l if $j = i$, or the interference-to-noise ratio per unit transmit power if $j \neq i$. It is also assumed that each Tx in the network is subject to the maximum transmit power constraint, i.e., the total power transmitted over n_t transmit antennas and all m frequency bands should be less than or equal to P_{max}. Let $\mathbf{R}_i^l$ represent the covariance matrix of the interference plus noise observed at the i^{th} Rx node in the l^{th} frequency band. Assuming interference plus noise to be Gaussian distributed, it can be computed as:

$$\mathbf{R}_i^l = \sum_{\substack{j=1 \\ j \neq i}}^{N} \rho_{ji}^l \mathbf{H}_{ji}^l \mathbf{Q}_j^l \mathbf{H}_{ji}^{l\dagger} + \mathbf{I}. \tag{1}$$

B. Capacity of a Single Band Multiuser MIMO Network

To begin, we examine the capacity of a single MIMO link in a SBMM network. Since $m = 1$ in this case, we drop the superscript l in all the variables defined above for simplicity. As discussed in [14], [17], the information theoretic capacity of this single MIMO link can be computed as $C_i = \log_2 \det(\mathbf{I} + \rho_{ii}\mathbf{R}_i^{-1}\mathbf{H}_{ii}\mathbf{Q}_i\mathbf{H}_{ii}^\dagger)$. Now since $\mathbf{Q}_i \succ \mathbf{0}$, it can be expressed as $\mathbf{Q}_i = \mathbf{U}_i\Lambda_i\mathbf{U}_i^\dagger$, where Λ_i is the diagonal matrix of the eigenvalues of $\mathbf{Q}_i$ and $\mathbf{U}_i$ is the unitary matrix with columns consisting of the eigenvectors of $\mathbf{Q}_i$. Defining $\widehat{\mathbf{H}}_{ii} = \mathbf{H}_{ii}\mathbf{U}_i$, the capacity expression can be written as $C_i = \log_2 \det(\mathbf{I} + \rho_{ii}\mathbf{R}_i^{-1}\widehat{\mathbf{H}}_{ii}\Lambda_i\widehat{\mathbf{H}}_{ii}^\dagger)$. As $\mathbf{Q}_i \succ \mathbf{0}$, it leads to the following two very important properties which are instrumental in the further simplification of the problem formulation:

1) The distributions of $\widehat{\mathbf{H}}_{ii}$ and $\mathbf{H}_{ii}$ are same [14].
2) All the eigenvalues of $\mathbf{Q}_i$, i.e. all the diagonal elements of Λ_i, are real and positive.

Due to these properties, it is sufficient to consider Λ_i instead of $\mathbf{Q}_i$ in the further discussion. We now define $\widetilde{\mathbf{H}}_{ii} = \mathbf{R}_i^{-1/2}\widehat{\mathbf{H}}_{ii}$ and the expression to find capacity is further simplified to:

$$C_i = \log_2 \det(\mathbf{I} + \rho_{ii}\widetilde{\mathbf{H}}_{ii}\Lambda_i\widetilde{\mathbf{H}}_{ii}^\dagger). \tag{2}$$

To maximize the sum of the capacities of this N-user SBMM network, we need to find the optimal real diagonal matrices Λ_i for all the N SUs. The problem can be mathematically formulated as:

$$\begin{array}{ll} \max & \sum_{i=1}^N C_i \\ s.t. & C_i = \log_2 \det(\mathbf{I} + \rho_{ii}\widetilde{\mathbf{H}}_{ii}\Lambda_i\widetilde{\mathbf{H}}_{ii}^\dagger) \\ & \widetilde{\mathbf{H}}_{ii} = \mathbf{R}_i^{-1/2}\widehat{\mathbf{H}}_{ii} \\ & \mathbf{R}_i = \sum_{\substack{j=1 \\ j\neq i}}^N \rho_{ji}\widehat{\mathbf{H}}_{ji}\Lambda_j\widehat{\mathbf{H}}_{ji}^\dagger + \mathbf{I} \\ & Tr\{\Lambda_i\} \leq P_{max}, \text{diag}\{\Lambda_i\} \succeq \mathbf{0}, 1 \leq i \leq N. \end{array} \tag{3}$$

C. Incorporating DSA in Optimal Multiuser MIMO

Using the framework developed in the previous subsections, we now incorporate DSA in the multiuser MIMO network and mathematically formulate the problem of finding the maximum capacity of the resulting MBMM network. Here also, we begin our discussion by analyzing the capacity of a single MIMO link which can be computed as $C_i = \sum_{l=1}^m \log_2 \det(\mathbf{I} + \rho_{ii}^l(\mathbf{R}_i^l)^{-1}\mathbf{H}_{ii}^l\mathbf{Q}_i^l\mathbf{H}_{ii}^{l\dagger})$. As explained in the previous subsection, $\mathbf{Q}_i^l$ being positive semidefinite, i.e. $\mathbf{Q}_i^l \succ \mathbf{0}$, can be expressed as $\mathbf{Q}_i^l = \mathbf{U}_i^l\Lambda_i^l\mathbf{U}_i^{l\dagger}$. Defining $\widehat{\mathbf{H}}_{ii}^l = \mathbf{H}_{ii}^l\mathbf{U}_i^l$, the capacity of the single MIMO link in this case can be written as $C_i = \sum_{l=1}^{m} \log_2 \det(\mathbf{I} + \rho_{ii}^l (\mathbf{R}_i^l)^{-1} \widehat{\mathbf{H}}_{ii}^l \Lambda_i^l \widehat{\mathbf{H}}_{ii}^{l\dagger})$. We simplify the expression by defining $\widetilde{\mathbf{H}}_{ii}^l = (\mathbf{R}_i^l)^{-1/2} \widehat{\mathbf{H}}_{ii}^l$. The single link capacity can now be expressed as

$$C_i = \sum_{l=1}^{m} \log_2 \det(\mathbf{I} + \rho_{ii}^l \widetilde{\mathbf{H}}_{ii}^l \Lambda_i^l \widetilde{\mathbf{H}}_{ii}^{l\dagger}). \tag{4}$$

Equation (4) can be further simplified by defining the following higher dimensional matrices:

$$\widetilde{\Lambda}_i = \begin{bmatrix} \Lambda_i^1 & 0 & \dots & 0 \\ 0 & \Lambda_i^2 & \dots & 0 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \dots & \Lambda_i^m \end{bmatrix}, \tag{5}$$

$$\widetilde{\mathbf{H}}_{ii} = \begin{bmatrix} \sqrt{\rho_{ii}^1} \widetilde{\mathbf{H}}_{ii}^1 & 0 & \dots & 0 \\ 0 & \sqrt{\rho_{ii}^2} \widetilde{\mathbf{H}}_{ii}^2 & \dots & 0 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \dots & \sqrt{\rho_{ii}^m} \widetilde{\mathbf{H}}_{ii}^m \end{bmatrix}. \tag{6}$$

Using these matrices, (4) can be written as

$$C_i = \log_2 \det(\mathbf{I} + \widetilde{\mathbf{H}}_{ii} \widetilde{\Lambda}_i \widetilde{\mathbf{H}}_{ii}^{\dagger}). \tag{7}$$

To maximize the sum of the capacities of all the SUs in the present system, we need to find m optimal diagonal matrices Λ_i^l for each SU or in other words find the optimal $n_t m$ diagonal elements of Λ_i for each SU. The mathematical formulation of this optimization problem is as follows:

$$\begin{aligned} \max \quad & \textstyle\sum_{i=1}^{N} C_i \\ s.t. \quad & C_i = \log_2 \det(\mathbf{I} + \widetilde{\mathbf{H}}_{ii} \widetilde{\Lambda}_i \widetilde{\mathbf{H}}_{ii}^{\dagger}) \\ & \widetilde{\Lambda}_i,\ \widetilde{\mathbf{H}}_{ii} \text{ defined by equations } (5),\ (6) \\ & Tr\{\widetilde{\Lambda}_i\} \leq P_{max}, \text{diag}\{\widetilde{\Lambda}_i\} \succeq \mathbf{0}, 1 \leq i \leq N. \end{aligned} \tag{8}$$

Comparing (3) and (8), we can easily conclude that the mathematical formulation of maximizing the system capacity of the MBMM network is exactly same as that of the SBMM network. This fact allows us to extend all the results and algorithms proposed in the literature for the latter. It is however important at this point to note that the computational complexity of (8) is roughly m times of that of (3). This is because in (3), optimization is to be performed over the eigenvalues of just one signal covariance matrix for each SU, whereas in (8), optimization has to be performed over m such covariance matrices (for each SU). Further, this optimization problem is shown to be neither a concave nor a convex problem and hence it is difficult to compute the globally optimum solution for this problem analytically [14]. Even the global optimization algorithms proposed to solve (3) are not easy to extend to the present problem because of their slow convergence and the enormous complexity of the problem especially for a large number of users and/or frequency bands. One such algorithm proposed in [17] uses the reformulation linearization technique coupled with the branch and bound framework to arrive at the global optimal solution. We are currently working on extending this algorithm to the current problem with suitable modifications, especially in the complexity aspects.

The basic idea of this algorithm is to construct a linear programming (LP) relaxation for the original nonlinear problem. The LP is then used to compute the global upper bound for the original nonlinear problem. This relaxed solution can either be a feasible or non-feasible solution to the original problem. If this solution is not feasible, it can be used as a starting point to find a feasible solution to the original nonlinear problem. This feasible solution will provide a global lower bound to the problem. Once the upper and lower bounds are computed, the branch and bound procedure can be applied to tighten the bounds and eventually arrive at a near optimal solution within the given approximation error. In addition to this, some other interesting non-convex nonlinear programming algorithms like interior point algorithms and sequential quadratic programming algorithms are also worth investigating in the light of the current problem [21], [22].

The mathematical framework developed above provides maximum achievable system capacity in the given channel conditions when DSA is incorporated in the multiuser MIMO network. As explained above, the problem of maximizing the capacity is reduced to finding the optimal eigenvalues of the input symbol covariance matrices of the SUs in each available frequency band. However, in practice, the optimal solution may not be practical. It is therefore necessary to investigate the capacity of multiband multiuser systems employing sub-optimal MIMO techniques to identify the techniques which approximate the optimal capacity in the given channel conditions. This issue is discussed in detail in the next section in the light of generalized beamforming.

IV. Incorporating DSA with Optimal Beamforming

In this section, we investigate the problem of incorporating DSA in the multiuser MIMO system where the Tx unit and Rx unit of each SU employs beamforming. The goal in optimal beamforming is to find the optimal frequency allocation and the optimal Tx and Rx beamforming weights to maximize the sum of the capacities of all the SUs. It is important to note here that optimal beamforming in cognitive MIMO exploits only a single channel mode for transmission in each frequency band and other modes remain unused. As discussed later in this section, this constraint can be easily incorporated in the general MIMO capacity optimization problem (8) to derive the mathematical formulation of a general beamforming capacity problem.

We start the problem formulation by defining the new variables required. Let $\mathbf{b}_{T_i}^l \in C^{1 \times n_t}$ and $\mathbf{b}_{R_i}^l \in C^{1 \times n_r}$ be the beamforming weights of the Tx and Rx, respectively, of the i^{th} SU in the l^{th} frequency band. Let f_i^l, for $i \in \{1, 2, \dots N\}$ and $l \in \{1, 2, \dots m\}$, be the power transmitted by the i^{th} user in the l^{th} frequency band. Total power transmitted over all the Tx antennas by the i^{th} SU is then $\sum_{l=1}^{m} f_i^l \leq P_{max}$. We now examine the transmit symbol covariance matrix of i^{th} SU in the l^{th} frequency band, which is given by:

$$\mathbf{Q}_i^l = f_i^l \begin{bmatrix} b_{T_i1}b'_{T_i1} & b_{T_i1}b'_{T_i2} & \dots & b_{T_i1}b'_{T_in_t} \\ b_{T_i2}b'_{T_i1} & b_{T_i2}b'_{T_i2} & \dots & b_{T_i2}b'_{T_in_t} \\ \vdots & & \ddots & \vdots \\ b_{T_in_t}b'_{T_i1} & b_{T_in_t}b'_{T_i2} & \dots & b_{T_in_t}b'_{T_in_t} \end{bmatrix}. \quad (9)$$

It is evident from (9) that all the columns of $\mathbf{Q}_i^l$ are linearly dependent and hence it has a single non-zero eigenvalue and the rest is null space. Since $\mathbf{Q}_i^l \succ \mathbf{0}$, it can be expressed as $\mathbf{Q}_i^l = \mathbf{U}_i^l \Lambda_i^l \mathbf{U}_i^{l\dagger}$, where Λ_i^l is a diagonal matrix of the eigenvalues of $\mathbf{Q}_i^l$ with only one of them being non-zero. Assuming that the norm of beamforming vector is one, it can be easily shown that the only nonzero eigenvalue of Λ_i^l is f_i^l. Physically, it establishes that each SU uses only one channel mode in each frequency band for transmission. To incorporate this fact in the general cognitive MIMO capacity optimization problem (8), let us define a diagonal matrix Γ_i^l with all the diagonal elements equals to zero except one, which is unity. To simplify the formulation, we now define a higher dimensional matrix $\widetilde{\Gamma}_i$ as:

$$\widetilde{\Gamma}_i = \begin{bmatrix} \Gamma_i^1 & 0 & \dots & 0 \\ 0 & \Gamma_i^2 & \dots & 0 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \dots & \Gamma_i^m \end{bmatrix}. \quad (10)$$

By incorporating $\widetilde{\Gamma}_i$ in (8), the mathematical formulation of the optimal beamforming capacity problem can be written as:

$$\begin{array}{ll} \max & \sum_{i=1}^N C_i \\ s.t. & C_i = \log_2 \det(\mathbf{I} + \widetilde{\mathbf{H}}_{ii}\widetilde{\Gamma}_i\widetilde{\Lambda}_i\widetilde{\mathbf{H}}_{ii}^\dagger) \\ & \widetilde{\Lambda}_i,\ \widetilde{\mathbf{H}}_{ii},\ \widetilde{\Gamma}_i \text{ defined by } (5),\ (6),\ (10) \\ & Tr\{\Gamma_i^l\} = 1, \mathrm{diag}\{\Gamma_i^l\} \in \{0,1\}, \forall\, i, l \\ & Tr\{\widetilde{\Lambda}_i\} \le P_{max}, \mathrm{diag}\{\widetilde{\Lambda}_i\} \succeq \mathbf{0}, 1 \le i \le N. \end{array} \quad (11)$$

After developing the required mathematical framework, we now present some numerical results in the next section.

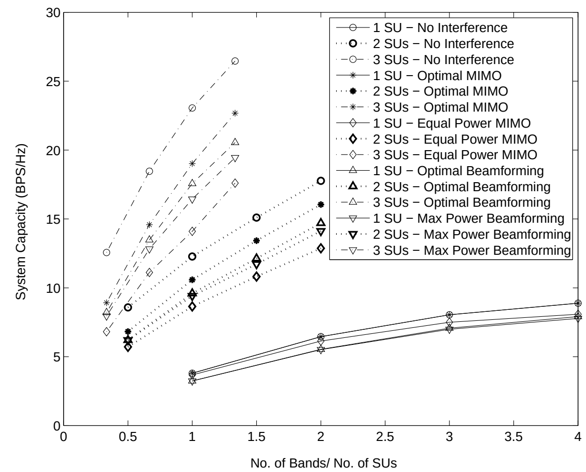

Fig. 2. Comparison of the system capacity results in a high interference scenario (MUI = 5).

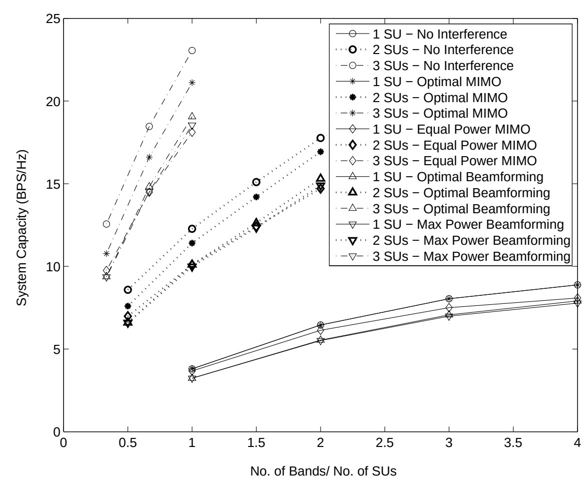

Fig. 3. Comparison of the system capacity results in a low interference scenario (MUI = 1).

V. NUMERICAL RESULTS

As discussed earlier, the problem of finding the optimal system capacity (sum rate) of a general MBMM network is neither a concave nor a convex problem and hence it is difficult to compute the globally optimum solution analytically. In this work, we employ a gradient based search method to solve both optimal MIMO and optimal beamforming capacity problems of a MBMM network. Though gradient-based search methods do not guarantee the globally optimal solution for non-convex problems, near optimal solutions can be reached by solving the problem multiple times with random starting points and combining the results. We are currently also developing a global optimization algorithm that would guarantee ϵ-optimal solution for such problems based on [17]. We will present this work in a future publication.

In addition to optimal MIMO and optimal beamforming, we also consider three special cases, viz., no interference, maximum equal power allocation and maximum power beamforming. In the no interference case, we assume that each SU is isolated from all the other SUs. This is a hypothetical case in the current setup and is just meant to upper bound the optimal capacity (sum-rate). In the maximum equal power allocation, we assume that each SU transmits P_{max}/mn_t power over each Tx antenna in each frequency band. Since this is not (in general) the optimal power allocation solution, this serves as a lower bound to the optimal MIMO capacity. In the maximum power beamforming case, we first choose an optimal channel mode in each frequency band and then transmit P_{max}/m power in each chosen channel mode. Since this is a special case of beamforming and is not the optimal solution in general, this will act as a lower bound to the optimal beamforming capacity.

The channel model is assumed to be a combination of large scale and small scale fading components. On the large scale, we assume that channel suffers from an exponential path loss with a path loss factor of 3 and from log-normal shadowing with a standard deviation of 1 dB. Small scale fading effects are modeled as Rayleigh distributed. Antennas at both the Tx and Rx of all the SUs are assumed to be independent in terms of small-scale fading but perfectly correlated in terms of log-normal shadowing. A high interference scenario is simulated by setting the MUI factor to be 5. The system capacity results for this case are presented in Fig. 2. Similarly, the system capacity results for a low interference scenario (MUI = 1) are presented in Fig. 3. We should remind the reader that the system capacity is defined as the sum of the link capacities of all the SUs in the network.

The importance of adopting an optimal MIMO transmission strategy can be gauged by comparing the optimal MIMO capacity results with the sub-optimal techniques such as beamforming and equal power MIMO. Numerical results presented in Fig. 2 and Fig. 3 highlight the importance of choosing the optimal transmit power and optimally distributing it over various channel modes in each band. The loss in system capacity due to interference can be evaluated by comparing the optimal MIMO capacity results with "no-interference" capacity results. As expected, the capacity loss is higher in high interference scenarios (Fig. 2) as compared to the low interference scenario (Fig. 3). The loss in system capacity also increases with the increase in number of SUs (due to increase in net interference).

Another important result can be drawn by comparing optimal beamforming capacity with the optimal MIMO capacity. Optimal beamforming capacity is observed to be closer to the optimal MIMO capacity in a high interference scenario than in a low interference scenario. This means that as the interference increases, it is optimal to use fewer channel modes for transmission. An analogous result for SBMM networks is discussed in [16], where it is proved that it is optimal to transmit over a single channel mode in a high interference scenario.

VI. Conclusions

In this paper, we have addressed the problem of incorporating DSA in a multiuser MIMO network. The mathematical framework developed is particularly helpful in computing the maximum achievable capacity (sum-rate) of a MBMM network. We have shown that this problem of capacity maximization is equivalent to finding the optimal eigenvalues of the input symbol covariance matrices of all SUs in each frequency band. We have further shown that the mathematical formulation of this optimization problem is quite similar to that of the well studied SBMM network. The importance of adopting an optimal MIMO transmission strategy is highlighted by comparing the optimal MIMO capacity results with the known suboptimal MIMO techniques, such as beamforming and maximum equal power transmission. We have further shown that it is optimal to transmit over fewer channel modes over each frequency band when interference increases. This problem of MBMM network sum-rate maximization, being non-convex and nonlinear, is difficult to solve analytically. We are currently developing a provably global optimization algorithm (based on [17]) to solve this problem and will include it in a future publication.

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