Distributed Caching in Device-to-Device Networks: A Stochastic Geometry Perspective

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Abstract—**Increasing spatio-temporal correlation in the data demand makes it attractive to cache popular content directly on the user devices so that it can be delivered on demand to the neighboring devices through device-to-device (D2D) communications. Due to the limited storage capacity, each device can only cache a part of the popular content resulting in a *distributed caching* network. With device locations modeled as a Poisson Point Process (PPP), we assume that the file of interest for a typical device is partitioned into several file portions with each device caching one of the portions randomly. Two equivalent viewpoints for the analysis of coverage probability and average delay in a distributed caching network are discussed for varying cache probabilities and device activity factors.**

Index Terms—**Distributed caching, D2D network, stochastic geometry, Poisson point process, coverage probability, delay.**

I. Introduction

Direct communication between proximate devices, termed device-to-device communications, is changing the way we envision, model and design cellular networks. While these networks have classically adopted a *content-agnostic* design approach, the presence of D2D links opens up a completely new possibility of *content-centric* operation. In particular, the devices can now cache popular content locally, which can then be delivered on demand to nearby devices. Owing to the limited storage capacities of the devices, all the popular content cannot be cached on each device, which means that a typical device may have to download content from multiple neighboring devices resulting in a *distributed* caching network. As the typical device tries to access content/file portions cached farther away from it, the number of potential dominant interferers increases, which presents a bottleneck in the content reception, possibly leading to a poor system performance. This degradation in the performance of D2D links has not been analyzed before and is the main focus of this work.

Related Work: Content duplication or caching reduces peak traffic by placing relevant content on various devices across the network during off-peak hours. The information theoretic formulation of this problem was introduced recently in [1] where an order-optimal coded caching scheme was also proposed. There has also been a lot of work on exploiting caching in small cell networks to alleviate backhaul load [2] and utilize content reuse through D2D communication [3]. The idea of *distributed caching* is studied in [4], where femtocell-like base stations called *helpers* form a wireless distributed caching network and an optimal caching strategy is proposed. Due to the irregular node locations and inherent randomness in the cache availability, *stochastic geometry* [5] turns out to be a very relevant tool for the analysis of these networks. Recent works on distributed caching in D2D networks based on stochastic geometry are primarily focused on finding the optimal content placement distribution. In [6], the authors find the optimal caching distribution that maximizes the probability of content delivery, assuming user demands are modeled as Zipf distribution. The authors in [7] evaluate the expected cost of obtaining a complete file from clients having cached parts of the file in their storage. The expected cost is shown to increase with distance between client and storage devices for both coded and uncoded allocation mechanisms.

Contributions: We develop a new model for a distributed caching D2D network in which the content of interest for the typical user is partitioned into several portions. Modeling the device locations as a Poisson Point Process (PPP), we derive easy-to-use expressions for the coverage probability and average delay experienced by a typical user in receiving each file portion. As expected, while the file portions that are cached closer to the device can be received fairly reliably, the D2D link performance degrades significantly while receiving farther file portions due to the presence of several potential dominant interferers. This exposes a performance tradeoff: while it is desirable to partition the content of interest into several portions due to storage constraints, it is not beneficial for the D2D link performance. Two different yet equivalent viewpoints, *file-type* based and *distance* based, are considered for the analysis of coverage probability and average delay for varying cache probabilities and device activity factors.

II. System Model

System Setup: Device locations are modeled as a homogeneous PPP Φ with intensity λ. In this paper, we consider a 2-file distributed caching system where each device has either file A or file B cached independently with probabilities $p_A = p$ and $p_B = 1 - p$. Independent thinning of the original PPP Φ results in two independent PPPs, Φ_A for file A and Φ_B for file B. See Fig. 1 for an illustration of this 2-file distributed caching setup with caching probabilities $p_A = 0.3$ and $p_B = 0.7$. As evident from the figure, file B is cached on more devices owing to its higher cache probability compared to file A. This setup can also be extended similarly to a K-file distributed caching case. For this system model, we are interested in finding the coverage and average delay experienced while receiving each file type. Conditioned on the serving link, each interferer is assumed to be active independently with probability q. This activity factor captures the fact that all the devices in the network may not always be active.

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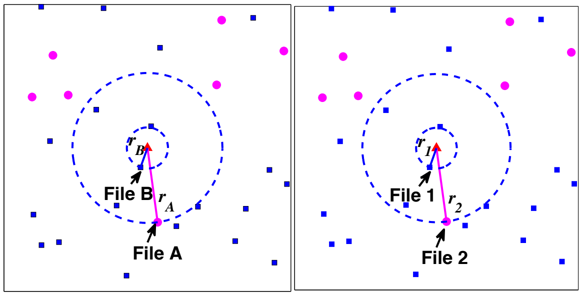

Fig. 1. Two-file distributed caching setup ($p_A = 0.3$ and $p_B = 0.7$). *(left)* File-type based viewpoint. *(right)* Distance-based viewpoint.

Channel Model: For the wireless channels, we assume power-law pathloss with exponent $\alpha > 2$ and Rayleigh fading. Without loss of generality, we assume that the D2D receiver of interest is located at origin. The received power at this receiver from a device located at $x \in \Phi$ is

$$P = P_t h_x \|x\|^{-\alpha}, \tag{1}$$

where P_t is the transmit power, $h_x \sim \exp(1)$ models Rayleigh fading, and $\|x\|^{-\alpha}$ is the pathloss. To define the interference power, we need an additional indicator random variable t_y, which takes value 1 with probability q and 0 otherwise. For this setup, the received signal to interference ratio (SIR) at the typical device (placed at the origin) is

$$\mathtt{SIR} = \frac{P_t h_x \|x\|^{-\alpha}}{\sum\limits_{y \in \Phi \setminus \{x\}} t_y P_t h_y \|y\|^{-\alpha}} = \frac{h_x \|x\|^{-\alpha}}{\sum\limits_{y \in \Phi \setminus \{x\}} t_y h_y \|y\|^{-\alpha}}. \tag{2}$$

We consider out-of-band D2D due to which the interference from the cellular network does not show up in the received SIR. The network is assumed to be interference-limited, which allowed us to ignore thermal noise in comparison to the interference power. This is usually the case in dense deployments where such a D2D network would be most relevant.

III. Coverage Probability Analysis

A device engaged in D2D communication is said to be in coverage if the received SIR at that device is greater than a coding and modulation specific threshold T. For Rayleigh fading, it is well known that the coverage probability, $p_c = P(\mathtt{SIR} > T)$, can be simply expressed in terms of Laplace transform of interference I as follows [5]:

$$p_c = \mathbb{P}(h_x > TI\| x \|^{\alpha}) \overset{(a)}{=} \mathcal{L}_I(T\| x \|^{\alpha}), \tag{3}$$

where (a) follows from the fact that $h_x \sim \exp(1)$. In this section, we discuss two different viewpoints for evaluating the coverage probability of obtaining the two files of interest.

A. File-Type based Viewpoint

In a file-type based viewpoint, the coverage probability of obtaining file of each type (A or B) is determined individually by connecting to the closest device that caches the file of that type. Fig. 1 (*left*) depicts this file-type based viewpoint, with r_A and r_B denoting the distances of the typical user to the closest device with file A and B, respectively. These two devices will act as *serving devices* to deliver files A and B.

Coverage probability of file A: Let us consider a scenario where the typical user tries to obtain file A cached in the network and connects to the closest device located at distance r_A. Since this device is simply the closest device of the PPP Φ_A of intensity $p_A\lambda$, the distribution of R_A can be found from its null probability as $f_{R_A}(r_A) = 2\pi p_A \lambda r_A e^{-p_A\lambda\pi r_1^2}$ [5].

The coverage probability of obtaining file A can be determined by considering interference from inside and outside the circle of radius r_A separately. We denote the outside and inside interference powers as I_{out} and I_{ins}, respectively.

Lemma 1. *The Laplace transform of interference at a typical user from all active devices outside the circle of radius r_A in a PPP of intensity λ and activity factor q is given by:*

$$\mathcal{L}_{I_{\text{out}}}(T{r_A}^{\alpha}) = \exp(-\pi q \lambda {r_A}^2 \rho_1(T,\alpha)) \tag{4}$$

$$\textit{where} \qquad \rho_1(T,\alpha) = T^{2/\alpha} \int\limits_{T^{-2/\alpha}}^{\infty} \frac{1}{1+u^{\alpha/2}}\,\mathrm{d}u. \tag{5}$$

Proof. This result follows directly from Theorem 2 in [8] for the no-noise case by observing that the interferers lying outside a circle of radius r_A with device activity factor q is equivalent to a PPP of intensity $q\lambda$ (outside the circle). ∎

The inside interference field is also handled in a similar way by observing that the all devices inside the circle of radius r_A by definition cache only file type B and hence constitute a PPP of intensity $(1 - p_A)\lambda$ with an activity factor q.

Lemma 2. *The Laplace transform of interference at the typical user from devices located inside the circle of radius r_A in a PPP of intensity λ and activity factor q is given by:*

$$\mathcal{L}_{I_{\text{ins}}}(T{r_A}^{\alpha}) = \exp(-\pi(1-p_A) q \lambda {r_A}^2 \rho_2(T,\alpha)) \tag{6}$$

$$\textit{where} \qquad \rho_2(T,\alpha) = T^{2/\alpha} \int\limits_{0}^{T^{-2/\alpha}} \frac{1}{1+u^{\alpha/2}}\,\mathrm{d}u. \tag{7}$$

Using the expressions of outside and inside interference from Lemma 1 and 2 respectively and exploiting the fact that I_{out} and I_{ins} are independent random variables, we multiply them to obtain the Laplace transform of total interference and hence the coverage probability of obtaining the closest file A.

Theorem 1. *The coverage probability of obtaining file A cached with probability p_A in a PPP network with intensity λ and activity factor q is*

$$\begin{aligned} p_c^{(A)} &= \int_0^{\infty} \mathcal{L}_{I_{\text{out}}}(T{r_A}^{\alpha}) \mathcal{L}_{I_{\text{ins}}}(T{r_A}^{\alpha}) f_{R_A}(r_A)\,\mathrm{d}r_A \\ &= \frac{p_A}{p_A + q[\rho_1(T,\alpha) + (1-p_A)\rho_2(T,\alpha)]} \end{aligned} \tag{8}$$

By symmetry, the coverage probability of obtaining file B (from the closest devices that caches it), $p_c^{(B)}$, is obtained by using its caching probability p_B in Theorem 1.

B. Distance based Viewpoint

A distance-based viewpoint of the coverage probability discussed in the current section provides a different perspective of the distributed caching problem by emphasizing on the proximity of each file type to the typical user. For this viewpoint, we label the closest file cached to the user (can be file A or B) as *file 1* and the file type cached "farther" from the typical user as *file 2*. Fig. 1 (*right*) depicts this distance-based viewpoint, with r_1 and r_2 denoting the distance of the typical user to the device with file 1 and 2 respectively. This viewpoint of the cached network leads to two possible scenarios as defined below.

$\mathcal{X}$: File A is file 1 (occurs with probability p_A).

$\mathcal{Y}$: File B is file 1 (occurs with probability p_B).

The distribution of R_1, the distance of the device having file 1 from the typical user, is determined from the null probability of the PPP Φ of intensity λ as $f_{R_1}(r_1) = 2\pi\lambda r_1 e^{-\lambda\pi r_1^2}$ [5]. Conditioned on a certain file type located at r_1, the distribution of R_2 is dictated by the presence of no devices caching the other file type between r_1 and r_2 and is given as

$$f_{R_2|R_1}(r_2|r_1) = \begin{cases} 0 & r_2 < r_1 \\ 2\lambda_2\pi r_2 e^{-\lambda_2\pi(r_2^2 - r_1^2)} & r_2 > r_1 \end{cases}$$

where $\lambda_2 = p_A\lambda$ with probability p_A and $p_B\lambda$ otherwise.

Coverage probability of file 1: Assuming that the typical user connects to the device located at distance r_1 (containing file 1), the coverage probability is determined by computing the Laplace transform of interference from devices located outside a circle of radius r_1 constituting a PPP of intensity λ and activity factor q. This is equivalent to considering just the outside interference ($p_A = 1$) in Theorem 1.

Corollary 1. *The coverage probability of obtaining the closest file in a PPP (file 1) of intensity λ and activity factor q is*

$$P_c^{(1)} = \frac{1}{1 + q\rho_1(T, \alpha)} \tag{9}$$

Coverage probability of file 2: Having obtained the closest file portion (file 1) located at r_1, the user now tries to obtain file 2 located at a distance r_2. For the coverage probability analysis of file 2, we just focus on subcase $\mathcal{Y}$ (file A is file 2). Subcase $\mathcal{X}$ can be handled in a similar way. The total coverage probability of obtaining file 2 is derived by applying total probability theorem to the two subcases as

$$P_c^{(2)} = p_A P_c^{(X)} + p_B P_c^{(Y)}, \tag{10}$$

where $P_c^{(X)}$ and $P_c^{(Y)}$ denote the conditional coverage probability of obtaining file 2 in subcases $\mathcal{X}$ and $\mathcal{Y}$, respectively. For subcase $\mathcal{Y}$, file B is located at the closest device of the PPP at distance r_1 and file A is located at a device farther away at distance r_2. Conditioned on r_1 and r_2, the coverage probability of obtaining file 2 in subcase $\mathcal{Y}$ can be determined by dividing the total interference field into three regions as described below.

I_1: Singleton interference from device $x \in \Phi_B$ at distance r_1

$$I_1 = t_x h_x r_1^{-\alpha} \tag{11}$$

I_2: Interference from all devices with file B except the singleton $\{x\}$ at distance r_1

$$I_2 = \sum_{y \in \Phi_B \setminus \{x\}} t_y h_y \|y\|^{-\alpha} \tag{12}$$

I_3: Interference from all devices with file A except the serving device $y \in \Phi_A$ at distance r_2

$$I_3 = \sum_{z \in \Phi_A \setminus \{y\}} t_z h_z \|z\|^{-\alpha} \tag{13}$$

Lemma 3. *(Interference from singleton set) The Laplace transform of interference at a typical user from the closest device in the PPP Φ (located at distance r_1) with the serving device at r_2 is given by:*

$$\mathcal{L}_{I_1}(Tr_2{}^{\alpha}) = (1-q) + \frac{q}{1 + T(\frac{r_2}{r_1})^{\alpha}} \tag{14}$$

Proof. By definition, the Laplace transform of interference is

$$\mathcal{L}_{I_1}(s) = \mathbb{E}[e^{-st_x h_x r_1^{-\alpha}}] \stackrel{(a)}{=} (1-q) + q\mathbb{E}[e^{-sh_x r_1^{-\alpha}}] \stackrel{(b)}{=} (1-q) + \frac{q}{1 + sr_1^{-\alpha}},$$

where (a) follows from the fact that the interferer $x \in \Phi_B$ located at r_1 is active with a probability q, and (b) results from $h_x \sim \exp(1)$. The final result now follows by substituting $s = Tr_2^{\alpha}$.

Lemma 4. *The Laplace transform of interference at a typical user from devices cached with file B outside a radius r_1 with the serving device at r_2 (subcase $\mathcal{Y}$) is given by:*

$$\mathcal{L}_{I_2}(Tr_2{}^{\alpha}) = \exp\left(-2\pi q p_B \lambda \int_{r_1}^{\infty} \left(\frac{T}{T + (\frac{r}{r_2})^{\alpha}}\right) r \,\mathrm{d}r\right). \tag{15}$$

Proof. This result also follows from Theorem 2 in [8] for the no noise case by observing that there are no interferers cached with file B lying inside a circle of radius r_1 and that the interferers lying outside r_1 with device activity factor q form an equivalent PPP of intensity $p_B q\lambda$. ∎

Interference from all devices caching file A outside the circle of radius r_2 with the serving device at r_2 is handled exactly as the outside interference from Lemma 1 and hence the Laplace transform of interference can be represented as

$$\mathcal{L}_{I_3}(Tr_2{}^{\alpha}) = \exp(-\pi p_A q \lambda r_2{}^2 \rho_1(T, \alpha)). \tag{16}$$

Using the expressions of interference from Lemmas 3 and 4 and equation (16), and exploiting the fact that all interference powers are independent random variables, we combine them to obtain the total interference and hence the coverage probability of obtaining file 2 in subcase $\mathcal{Y}$.

Theorem 2. *The coverage probability of obtaining file 2 at the typical device in a PPP of intensity λ and activity factor q for subcase $\mathcal{Y}$ is*

$$P_c^{(Y)} = \int_0^{\infty}\int_{r_1}^{\infty} \mathcal{L}_{I_1}(Tr_2{}^{\alpha})\mathcal{L}_{I_2}(Tr_2{}^{\alpha})\mathcal{L}_{I_3}(Tr_2{}^{\alpha}) f_{R_2|R_1}(r_2|r_1) f_{R_1}(r_1)\,\mathrm{d}r_2\,\mathrm{d}r_1 \tag{17}$$

IV. Average Delay

The average delay (also called *local delay*) is defined as the mean number of time slots required by the transmitter to successfully transmit to the receiver. Conditioned on the realization of a PPP Φ, the conditional success probability $p_{c|\Phi} = \mathbb{P}(\mathtt{SIR} > T|\Phi)$ is given by the event that the receiver is successfully connected to its assigned transmitter. If the receiver fails to decode the file, it is retransmitted in the next scheduled transmission slot. Hence, the distribution of the conditional local delay is geometric with mean $p_{c|\Phi}$ [9], taking the expectation of which over different PPP realizations yields the average delay. The average delay is hence given by

$$\mathbb{E}[D_I] = \mathbb{E}_\Phi\left[\frac{1}{\mathbb{P}(\mathtt{SIR} > T|\Phi)}\right] = \mathbb{E}_\Phi\left[\frac{1}{\mathcal{L}_I(s|\Phi)}\right] \quad (18)$$

A. File-Type based Analysis

The two different viewpoints considered in the analysis of coverage probability are also examined for the case of average delay. The mean number of time slots required to successfully transmit from the closest device with file A to the typical device is denoted as $\mathbb{E}[D_A]$ and derived in Lemma 5.

Lemma 5. *The average delay in communicating with the closest device with file A in a PPP of intensity λ and activity factor q is*

$$\mathbb{E}[D_A] = \frac{p_A}{p_A - q[\rho_3(T,\alpha) + (1-p_A)\rho_4(T,\alpha)]},$$

where $$\rho_3(T,\alpha) = T^{2/\alpha}\int_{T^{-2/\alpha}}^{\infty} \frac{1}{1-q+u^{\alpha/2}}\,\mathrm{d}u, \quad (19)$$

and $$\rho_4(T,\alpha) = T^{2/\alpha}\int_{0}^{T^{-2/\alpha}} \frac{1}{1-q+u^{\alpha/2}}\,\mathrm{d}u. \quad (20)$$

Proof. By definition of the average delay (18), we have

$$\mathbb{E}[D_A] \quad = \quad \mathbb{E}_\Phi\left[\frac{1}{\mathcal{L}_{I_{\text{out}}}(Tr_A{}^\alpha|\Phi)\mathcal{L}_{I_{\text{ins}}}(Tr_A{}^\alpha|\Phi)}\right] \quad (21)$$

The result follows directly from Theorem 1 in [10] using suitable limits of integration for the outside and inside interference. ■

By symmetry, the average delay in communicating with the closet device with file B, $\mathbb{E}[D_B]$, can be obtained by using its corresponding caching probability p_B in Lemma 5.

B. Distance based Analysis

The distance-based analysis gives the mean number of time slots required for successfully communicating with the closest devices caching files 1 and 2, which are denoted by $\mathbb{E}[D_1]$ and $\mathbb{E}[D_2]$ respectively. $\mathbb{E}[D_1]$ is obtained by considering only the outside interference in Lemma 5 ($p_A = 1$) and is given next.

Corollary 2. *The average delay in communicating with the closest device with file 1 in a PPP of intensity λ and activity factor q is*

$$\mathbb{E}[D_1] = \frac{1}{1-q\rho_3(T,\alpha)}.$$

Theorem 3. *The average delay in communicating with the closest device cached with file 2 for a given λ and q is*

$$E[D_2] = pE[D_2]^{(X)} + (1-p)E[D_2]^{(Y)} \quad (22)$$

where

$$E[D_2]^{(Y)} = \int_0^\infty\int_{r_1}^\infty \frac{1}{1-q+\frac{q}{1+sr_1^{-\alpha}}} \exp\left(2\pi p_B q\lambda\int_{r_1}^\infty \frac{r\,\mathrm{d}r}{1-q+\frac{r^\alpha}{s}}\right)\exp(\pi p_A q\lambda r_2^2\rho_3(T,\alpha)) f_{R_2|R_1}(r_2|r_1)f_{R_1}(r_1)\,\mathrm{d}r_2\,\mathrm{d}r_1.$$

Proof. See Appendix C. ■

As evident in Fig. 4, we have the following intuitive relation:

$$\mathbb{E}[D_1] + \mathbb{E}[D_2] = \mathbb{E}[D_A] + \mathbb{E}[D_B]. \quad (23)$$

The above equality demonstrates the equivalence of the two viewpoints undertaken in this paper. The file-type based viewpoint provides us with simpler expressions of average delay for each file type (Lemma 5) compared to the more complex results in the distance-based viewpoint (Theorem 3). However, the distance based viewpoint gives a better handle on the worst case coverage probability and average delay i.e. when the file of interest is cached “farther” away.

V. Results and Discussion

1) Effect of caching probability: The coverage probability of a file increases with its caching probability in the network. This is evident from Fig. 2, where the coverage probability of obtaining the files of interest are plotted for the two different viewpoints. As expected, file B with its higher caching probability of $p_B = 0.7$ has a better coverage than file A with $p_A = 0.3$. The figure also depicts the sharp decrease in the coverage probability of obtaining the farther cached file 2 compared to obtaining file 1 (around 17x lower at 0 dB). Similar trends can be observed for the case of average delay, with the higher cached file B requiring a lower number of time slots on average for successful transmission compared to the lower cached file A.

2) Effect of activity factor: Fewer the number of devices active in the network, lesser is the interference and better the coverage. This is the classic tradeoff between more aggressive frequency reuse (and hence higher throughput) and higher interference power. As evident from Fig. 3, the plots indicate a significant increase in coverage probability with decrease in the activity factors.

3) Critical SIR threshold: The mean number of time slots required for a successful file transmission increases with SIR threshold resulting in unsuccessful transmissions after a certain SIR threshold, defined as the cut-off threshold T_c. This is evident from Fig. 4 which also indicates the increasing cut-off threshold with the file's caching probability in the network. As was the case for coverage probability, it can be seen that lower delay values are encountered for lower activity factors due to a reduced interference power.

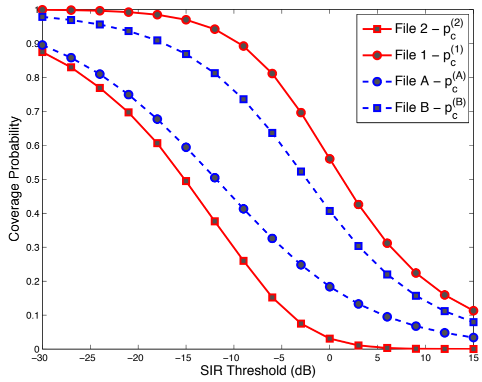

Fig. 2. Coverage probability in distance-based viewpoint vs. file-type based viewpoint ($p_A = 0.3$, $q = 1$ and $\alpha = 4$).

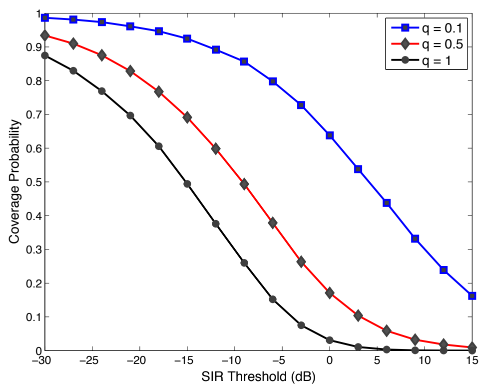

Fig. 3. Coverage probability of file 2 for different activity factors q ($p_A = 0.3$ and $\alpha = 4$).

VI. Conclusion

This paper deals with the modeling and analysis of a D2D network where distributed caching is employed. By discussing two equivalent viewpoints for the analysis of coverage probability and average delay, we showed diminishing coverage and large average delays for obtaining a file portion cached "farther" away. This showed the importance of caching files closer to the user of interest for better network performance. Also, the coverage probability and average delay improve with higher caching probabilities and lower activity factors.

Appendix

A. Proof of Theorem 3

Conditioned on r_1 and r_2, the average delay of obtaining file 2 is dictated by the average delay from three interference regions individually (I_1, I_2 and I_3) and is given as

$$E[D_2]^{(Y)} = \int_0^\infty \int_{r_1}^\infty \mathbb{E}[D_{I_1}]\mathbb{E}[D_{I_2}]\mathbb{E}[D_{I_3}] \, f_{R_2|R_1}(r_2|r_1) f_{R_1}(r_1)\,\mathrm{d}r_2\,\mathrm{d}r_1 \tag{24}$$

$$\mathbb{E}[D_{I_1}] = \mathbb{E}_\Phi\left[\frac{1}{1-q+q\mathbb{E}[e^{-sh_x r_1^{-\alpha}}]}\right] \overset{(a)}{=} \frac{1}{1-q+\frac{q}{1+sr_1^{-\alpha}}}$$

$$\mathbb{E}[D_{I_2}] = \mathbb{E}_\Phi\left[\prod_{y\in\Phi_B\setminus(\{x\}\cup B(0,r_1))} \frac{1}{1-q+q\mathbb{E}[e^{-sh_y\|y\|^{-\alpha}}]}\right]$$

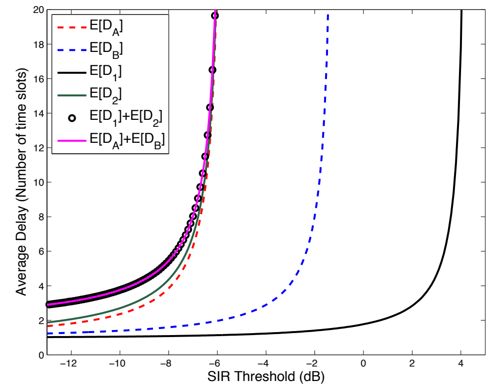

Fig. 4. Average delay for the file-type based and distance based viewpoints ($p_A = 0.3, q = 0.5, \alpha = 4$).

$$\overset{(b)}{=} \exp\left(-2\pi p_B\lambda\int_{r_1}^\infty\left(1-\frac{1}{1-q+\frac{q}{1+sr^{-\alpha}}}\right)r\,\mathrm{d}r\right)$$

$$= \exp\left(2\pi p_B q\lambda\int_{r_1}^\infty\frac{r\,\mathrm{d}r}{1-q+\frac{r^\alpha}{s}}\right)$$

$$\mathbb{E}[D_{I_3}] = \mathbb{E}_\Phi\left[\prod_{z\in\Phi_A\setminus B(0,r_2)}\frac{1}{1-q+q\mathbb{E}[e^{-sh_z\|z\|^{-\alpha}}]}\right]$$

$$\overset{(c)}{=} \exp\left(-2\pi p_A\lambda\int_{r_2}^\infty\left(1-\frac{1}{1-q+\frac{q}{1+sr^{-\alpha}}}\right)r\,\mathrm{d}r\right)$$

$$\overset{(d)}{=} \exp(\pi p_A q\lambda r_2^2\rho_3(T,\alpha))$$

where (a) follows from the fact that $h_x \sim \exp(1)$, (b) and (c) result from the PGFL of the PPP Φ while considering the activity factor q of the interferers, and (d) follows by rearranging a few terms and defining $\rho_3(T,\alpha)$ as in (20). The result now follows by substituting $s = Tr_2^\alpha$.

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